CISC 1400
Discrete Structures
Chapter 0
Introduction

Arthur G. Werschulz

Fordham University Department of Computer and Information Sciences

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Welcome to CISC 1400!

- A computer science course, seasoned with a soupçon of math.
- Or maybe a math course, tilted towards things a computer scientist needs to know.
- Counts towards the mathematical and computational reasoning requirement of the Fordham Core Curriculum.
- Required course for Computer Science and Information Science majors.
- Also used occasionally as an elective.

About your host

- Dr. Arthur G. Werschulz
- Office Hours: MTWR noon–1:00 or by appointment
- Office: LL 610D
- Phone: 212-636-6325
- Email: agw@dsm.fordham.edu
- Class email list: discrete@dsm.fordham.edu

Objective and desired outcomes

- Objectives:
  - To develop the necessary abstract reasoning abilities while learning to succeed in a mathematical and computer environment
  - Develop some of the math background needed in later CISC courses
- Desired outcomes:
  - Be able to analyze and understand common math notation
  - Be able to develop solutions to mathematical problems
  - Be able to use a well-defined methodology to reason about math
  - Be able to develop solution to multi-step reasoning problems

Available resources

Textbook Lyons *et al.*, *Fundamentals of Discrete Structures.* Second Edition, 2012.

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Website `http://www.dsm.fordham.edu/~agw/discrete`

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Instructor: He would love to help you out. Take advantage of office hours and email!

Things you really must know about

Attendance Really just short of mandatory. We are all busy people but I need to have you here for all class sessions. Unexcused absences or missing more than 4 classes will lower your course grade.

Homework Expect to spend approximately 6 hours each week on work. We'll discuss each day's homework at the next class session. So either know it, or be ready to ask about it!

Grading As follows:

- Participation: 10%
- Homework: 30%
  - Written homework: 20%
  - Computer project: 10%
- Midterm exam: 30%
- Final exam: 30%

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- **Academic integrity:** In short: the work you do should be your own. You are only allowed help from authorized sources or when I explicitly permit it. You should read Fordham's academic integrity policy to know all your rights and all the rules

What's discrete mathematics?

- **Continuous mathematics:** deals with objects that can take on a continuous (smooth) set of values (high school algebra, trigonometry, . . .)
- **Discrete mathematics:** deals with objects that can only assume distinct, separated values
  - Sequences, sets
  - Logic
  - Relations, functions
  - Counting, probability
  - Graphs
- Useful for modeling many real-world objects (e.g., the Internet)
- Especially useful for computer problem solving
- Very practical!

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- Can do certain *operations* on sets (union, intersection, complement, . . .)

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- $1, 2, 4, 8, \ldots 15$!!!!
- Why? Let $a_n = \frac{4}{3}n - \frac{1}{2}n^2 + \frac{1}{6}n^3$. Then

n	1	2	3	4	5	6
a_n	1	2	4	8	15	26

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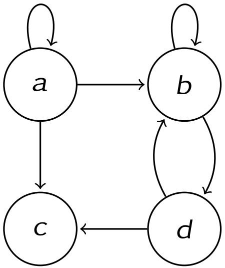
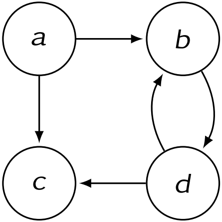
- Relational data bases: needed for e-commerce

Relations may represented by graphs



Visualizing relations with directed graphs



This graph could represent:

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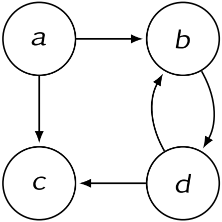
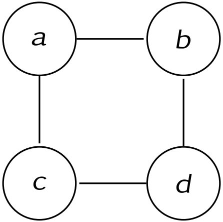


This graph could represent:

- Pairs of people, in which the first has sent an email to the second.
- Part of a street map.

Visualizing relations with undirected graphs



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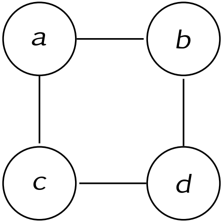

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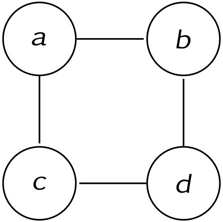
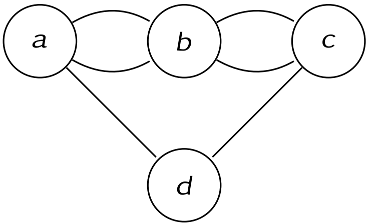


This graph could represent:

- Friendship within Facebook.
- Connections within LinkedIn.

Graph problems

Can you draw the picture



without lifting the pencil or retracing any part of the figure?

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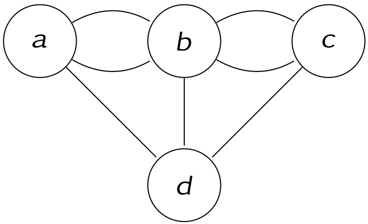

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- Engineering: how to connect five cities to via a highway with minimal cost?
- Scheduling: how to assign classes to classrooms so that minimal number of classrooms are used?

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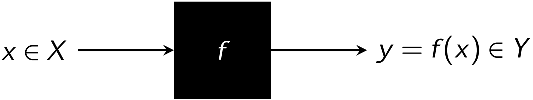

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$$x \in X \longrightarrow \boxed{f} \longrightarrow y = f(x) \in Y$$

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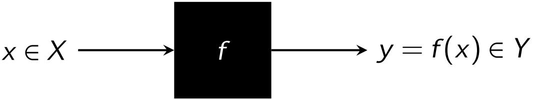

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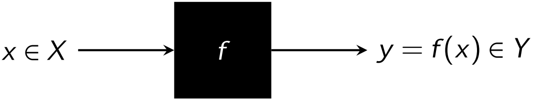

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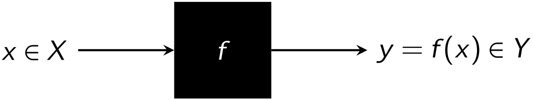

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- Examples of simple functions:
  - "Birth date of" is a function from people to calendar dates (but not vice versa!).
  - "Social security number" is a function from the set of people having SSNs to the set of assigned SSNs (and vice versa).

Our class: birthday remark

- Someone says:

  There are at least two students in the class that were born in the same season.

  Do you agree?

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- **Pigeonhole principle:** If you put m pigeons into n pigeonholes, where $m > n$, then there is a pigeonhole containing at least two pigeons.

Another pigeonhole principle example: choosing a pair of socks

- Suppose that you have three different kinds of socks.
- Suppose further that you shut your eyes and reach into your sock drawer.
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- Harder questions:
  - In how many ways can we elect a representative and an alternate?
  - In how many ways can we choose ...
    - a 2-person study group?
    - a 3-person study group?

Probability: How likely is something to happen?

- Suppose I choose one person randomly from the class. How likely are you to be chosen?

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  - What's the probability of winning New York State Lotto (pick 6 out of 59)?
  - What about MegaMillions or PowerBall?

Logic: A tool for problem solving

- Your friend tells you:

  If the birds are flying south and the leaves are turning, then it must be fall. Fall brings cold weather. The leaves are turning, but the weather is not cold. Therefore the birds are not flying south.

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- Do you agree with her?
- Is her argument valid? sound? (what's the difference)?

Let's analyze her argument

- Suppose the following are true:
  - If the birds are flying south and the leaves are turning, then it must be fall.
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Let′s analyze her argument

- Suppose the following are true:
  - If the birds are flying south and the leaves are turning, then it must be fall.
  - Fall brings cold weather.
  - The leaves are turning but the weather is not cold.
- Can one conclude "the birds are not flying south"?

Let's analyze her argument (cont'd)

- We'll do a "proof by contradiction".
  - Assume that the birds are flying south.

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  - But it's actually not cold!!

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  - Fall brings cold weather. So it must be cold.
  - But it's actually not cold!!
- Contradiction! So our assumption that the birds are flying south must be wrong. □

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- The *Sutherland-Hodgman polygon clipping algorithm*: speeds up the rendering of images for computer graphics and video game programs by removing objects that do not fall into the "camera's" field of view.
- Algorithms in machine learning, data analytics: getting a lot of attention these days.

Our list of topics:

- Sets
- Sequences
- Logic
- Relations
- Functions
- Counting
- Probability
- Algorithms
- Graph theory

Goals for this course

- Master the basics of discrete mathematics
- Develop mathematical and computational reasoning abilities
- Become more comfortable and confident with both mathematics and computation

Discrete mathematics is essential for computer problem solving

- Model real-world entity
  - Student records in a registration system $\rightarrow$ elements of a set
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- Develop/identify an algorithm solving a particular problem
  - Search for a student record (using ID number)
  - Query for a course having a particular prefix (e.g., "CISC")
  - Find shortest path in a graph (think Google Maps)

Discrete mathematics is essential for computer problem solving

- Model real-world entity
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  - Find shortest path in a graph (think Google Maps)
- Implement algorithm using a programming language that computers "understand"