CISC 1100—Structures of Computer Science
Tuesday 6 December 2011

PRACTICE FINAL EXAMINATION

Problem 1. (________/ 10 points)
Draw a Venn diagram that illustrates $(A \cup B)^{\complement}$.

Problem 2. (________/ 10 points)
Use a truth table to determine whether

$$(p \Rightarrow q) \equiv (\neg p \Rightarrow \neg q)$$

is a tautology (i.e., always true).

Problem 3. (______/ 15 points)
Let $S = \{1, 2, 3\}$. Consider the relation

$$r = \{(1, 1), (1, 2), (1, 3), (2, 3), (3, 3)\}$$

on S.

1. Explicitly state which of the "big five" properties are satisfied by this relation r. If you do not put either a `Y` or a `N` in the blank provided, then that part of the problem will be marked as being incorrect.

   - Reflexive? ____
   - Irreflexive? ____
   - Symmetric? ____
   - Antisymmetric? ____
   - Transitive? ____

2. Could r be a function from S to S? Explain why or why not.

Problem 4. (______/ 10 points)
Define a function $f : \mathbb{N} \rightarrow \mathbb{N}$ by

$$f(x) = 2x + 5 \qquad \forall x \in \mathbb{N}.$$

What is $(f \circ f)(x)$?

Problem 5. (________/ 10 points)
Let $f : \mathbb{R}^{\geq 0} \to \mathbb{R}^{\geq 9}$ be defined as

$$f(x) = 2x^2 + 9 \qquad \forall x \in \mathbb{R}^{\geq 0}.$$

This function is invertible. Find its inverse.

Problem 6. (________/ 10 points)
Compute each of the following.

1. $C(5, 0)$.

2. $C(5, 1)$.

3. $C(5, 2)$.

4. $C(1000, 1)$.

5. $C(1000, 1000)$.

Problem 7. (________/ 10 points)

One of my favorite pizzerias sells pizzas with the following toppings: pineapple, felafel, mushrooms, green peppers, and onions.

1. Taking account of all possible situations (i.e., you might decide to order any number of these toppings, or none at all), how many different kinds of pizzas can you order?

2. The pizzeria is running a post-exam special, where a customer can get up to two toppings for free. How many different kinds of such pizzas (i.e., with *at most* two toppings) are there?

Problem 8. (________/ 15 points)

In (five-card draw) poker, each player is dealt five cards out of a 52-card deck.

1. How many different poker hands are there?

2. In how many ways can one get four of a kind, i.e., a hand in which four of the five cards have the same denomination?

3. What is the probability of drawing four of a kind in five-card draw poker?

Problem 9. (________/ 10 points)

A red hat contains ten numbers (`1`, `2`, ..., `10`) and a blue hat contains ten letters (`A`, `B`, ..., `J`). You draw one item from each hat. You will win a prize if the number is less than or equal to 4 or the letter is an `A`, `B`, or `C` (or both occur). What is the probability that you win a prize?

Problem 10. (________/ 10 points)

How many distinguishable rearrangements can you make from the word `PARACELSUS`?