CISC 1100—Structures of Computer Science
Wednesday 5 October 2011

PRACTICE MIDTERM EXAMINATION (SOLUTIONS)

Problem 1. (________/ 15 points)
Consider the sequence

$$0, 3, 8, 15, 24, \ldots .$$

1. What is the next term in the sequence?

   The differences have the pattern $3, 5, 7, 9$***, and so the next difference will be*** 11***. Thus the next term will be*** $24 + 11 = 35$***.***

2. Determine the recursive formula for the sequence. (Don't forget the starting value!)

   Trying a few values, we see that

$$\begin{aligned} a_1 &= 3 = 0 + 3 = a_0 + \textbf{\textit{the second odd number}} \\ a_2 &= 8 = 3 + 5 = a_1 + \textbf{\textit{the second odd number}} \\ a_3 &= 15 = 8 + 7 = a_2 + \textbf{\textit{the third odd number}} \end{aligned}$$

   and so forth. In general,

$$a_n = a_{n-1} + \textbf{\textit{the }} (n+1)^{\textbf{\textit{st}}} \textbf{\textit{ odd number}}.$$

   Since the $(n+1)^{st}$ ***odd number is*** $2n + 1$***, we have***

$$\begin{aligned} a_n &= a_{n-1} + 2n + 1 \qquad \textbf{\textit{for }} n \geq 2 \\ a_1 &= 0 \end{aligned}$$

3. Determine the closed formula for the sequence.

   Since the terms increase linearly, we know that a_n ***is quadratic in*** n***. Comparing the values of the sequence with the values of*** n^2***, we have a table***

n	***1***	***2***	***3***	***4***	***5***	***6***
a_n	***0***	***3***	***8***	***15***	***24***	***35***
n^2	***1***	***4***	***9***	***16***	***25***	***36***

   Comparing the last two rows, we see that

$$a_n = n^2 - 1.$$

Problem 2. (________/ 5 points)
Express the sum

$$3 + 6 + 9 + 12 + 15 + 18$$

using sigma-notation.

Solution:

$$\sum_{j=1}^{6} 3j$$

Problem 3. (________/ 5 points)
Evaluate the sum

$$\sum_{i=1}^{5} (3i + 2)$$

Solution:

$$\begin{aligned}\sum_{i=1}^{5} (3i + 2) &= (3 \times 1 + 2) + (3 \times 2 + 2) + (3 \times 3 + 2) + (3 \times 4 + 2) + (3 \times 5 + 2) \\ &= 5 + 8 + 11 + 14 + 17 \\ &= 55\end{aligned}$$

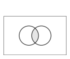
Problem 4. (________/ 10 points)
Draw Venn diagrams that illustrate following operations:

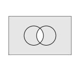
1. $A \cap B$.
2. $(A \cap B)^{\mathsf{C}}$.

(That's two different Venn diagrams.)

Solution:

and





Problem 5. (________/ 20 points)
Let

$$A = \{2, 3, 5, 7, 11\}$$
$$B = \{2, 4, 6, 8, 10\}$$
$$C = \{1, 3, 5, 7, 9\}$$

Determine the following:

1. $A \cap B$

   Solution: $\{2\}$

2. $(A \cap B) \cup C$

   Solution: $\{1, 2, 3, 5, 7, 9\}$

3. $(A \cup B) \cap C$

   Solution: $\{3, 5, 7\}$

4. $A - B$

   Solution: $\{3, 5, 7, 11\}$

5. $|\mathscr{P}(B)|$

   Solution: $2^5 = 32$

Problem 6. (________/ 10 points)

In a recent survey, 25% of the respondents said that we should raise taxes, 40% said that we should cut the Federal budget, and 55% said that we should do one or the other (perhaps both). What percentage of the respondents said that we should both raise taxes *and* cut the Federal budget?

For the sake of argument, suppose that there were 1000 people in the survey. Let T ***denote the set of respondents who said that we should cut taxes and let*** B ***denote the set of respondents who said that we should cut the Federal budget. Then*** $|T| = 25$, $|B| = 40$***, and*** $|T \cup B| = 55$***. By the inclusion/exclusion principle, we have***

$$|T \cup B| = |T| + |B| - |T \cap B|,$$

and thus

$$|T \cap B| = |T| + |B| - |T \cup B| = 25 + 40 - 55 = 10,$$

i.e., 10% say that we should both raise taxes **and** ***cut the Federal budget.***

Problem 7. (________/ 5 points)

Suppose that you (or somebody else) has proved that the propositional equivalence

$$p \wedge (\neg q \vee r) \equiv (p \wedge \neg q) \vee (p \wedge r)$$

is true. The duality principle tells us that the dual of this equivalence is also true. What is the dual of the equivalence given above?

Solution:

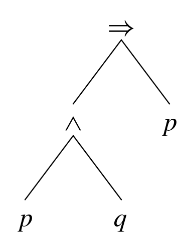
$$p \vee (\neg q \wedge r) \equiv (p \vee \neg q) \wedge (p \vee r)$$

Problem 8. (________/ 10 points)

Draw the parse tree of the expression

$$p \wedge q \Rightarrow p$$

Solution:



Problem 9. (________/ 15 points)

Let the variables f, s, and p stand for "the food is good", "the service is excellent", and "the price is high", respectively. Translate the following English sentences into propositional form.

1. Either the food is good or the service is excellent.

   Solution: $f \lor s$

2. The food is good and the service is excellent.

   Solution: $f \land s$

3. The food is good, but the service is not excellent.

   Solution: $f \land \neg s$***. If you prefer*** $f \land (\neg s)$***, that's okay too.***

4. Either the food is good and the service is excellent, or else the price is high.

   Solution: $f \land s \lor p$***. If you prefer*** $(f \land s) \lor p$***, that's okay too.***

5. If the price is high, then the food is good and the service is excellent.

   Solution: $p \Rightarrow f \land s$***. If you prefer*** $p \Rightarrow (f \land s)$***, that's okay too.***

Problem 10. (________/ 15 points)

Use a truth table to prove DeMorgan's Law

$$\neg(p \land q) \equiv (\neg p) \lor (\neg q)$$

Solution:

p	q	$p \land q$	$\neg(p \land q)$	$\neg p$	$\neg q$	$(\neg p) \lor (\neg q)$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T