CISC 5835—Algorithms for Data Analytics
Fall, 2019

Chapter 0 Homework

1. [5 points] For each of the following, indicate whether $f = O(g)$, $f = \Omega(g)$, or $f = \Theta(g)$). (Only one response per question: if the correct answer is Θ, don't write "O, Ω, Θ".)

(a) $f(n) = 2n - 5$, $g(n) = 14n + 17$.

(b) $f(n) = n^2 + 3n + 2$, $g(n) = 14n \log n$.

(c) $f(n) = n^{100}$, $g(n) = 0.001^n$.

(d) $f(n) = n^{1.0001}$, $g(n) = n \log n$.

(e) $f(n) = 2^n$, $g(n) = n!$.

2. [10 points] Let k be a positive integer and let

$$S_n = \sum_{j=1}^{n} j^k = 1^k + 2^k + \cdots + n^k \qquad \text{for any positive integer } n.$$

Show that $S_n = \Theta(n^{k+1})$.

Hints:

1. The proof does *not* use induction.
2. For the upper bound, note that $j \le n$ within the sum.
3. For the lower bound, look at the sum of the biggest $n/2$ elements, and note that $j \ge n/2$ within the sum.

3. [10 points] Let $\phi = \frac{1}{2}(1 + \sqrt{5})$, which is roughly 1.618. Show that for the nth Fibonacci number F_n, we have $F_n = \Theta(\phi^n)$.

Hints:

1. Note that $x = \phi$ is the positive solution of the polynomial equation $x^2 - x - 1 = 0$. (The other solution is $x = \frac{1}{2}(1 - \sqrt{5}) \doteq -0.618$, for what it's worth.)
2. Since this is a Θ-result:
   - You only need to show it for large enough n.
   - You need to show an upper bound (a O-result) and a lower bound (a Ω-result). Their proofs are similar.
3. Use the second principle of mathematical induction.