

### Chapter 3 Homework

In what follows, if you are asked to traverse a graph and you have a choice of vertices, you are to pick the one that is alphabetically first.

1. [20 points] Let  $G$  be the directed graph with vertices  $a, b, c, d, e, f$  and edges

$$(a, b), (b, c), (b, d), (c, a), (c, f), (d, e), (d, f), (e, f).$$

- (a) [5 points] Draw  $G$ . Your answer should be a *planar* representation, i.e., one in which no edges cross each other. You can do this by putting vertices  $a, b, d$ , and  $f$  in one row, and vertices  $c$  and  $e$  in a second row, underneath the first row.
- (b) [10 points] Do a depth-first search on  $G$ , starting at vertex  $a$ . Give the pre and post numbers of each vertex. Also, list the vertices in the order found by the DFS.
- (c) [5 points] Classify each edge as a tree edge, forward edge, back edge, or cross edge.

2. [15 points] Let  $G$  be the directed graph with vertices  $a, b, c, d, e, f$  and edges

$$(a, b), (a, c), (b, c), (b, d), (c, f), (d, e), (d, f), (e, f)$$

Run the DFS-based topological sorting algorithm on  $G$ . Make sure to list the vertices in the proper topological order, i.e., if  $f, e, d, c, b, a$  is a topological ordering of the vertices, say so explicitly.

**Note:** If you decide to draw the graph, use a planar representation. See the suggestion in Problem 1.

3. [15 points] Let  $G$  be the directed graph with vertices  $a, b, c, d, e, f$  and edges

$$(a, b), (b, c), (b, d), (c, a), (c, f), (d, e), (d, f), (e, f).$$

Run the strongly connected components algorithm on  $G$ , answering the following questions:

- (a) Which are source SCCs and which are sink SCCs?
- (b) Draw the metagraph.
- (c) Give a minimal set of edges that must be added to this graph to make it strongly connected.