CISC 5835—Algorithms for Data Analytics
Fall, 2019

Chapter 5 Homework

1. [10 points] Consider the weighted graph with vertex set $\{a,b,c,d,e,f\}$ and weighted edge set

$$(\{a,b\},1),(\{a,d\},1),(\{a,e\},1),(\{b,c\},2),(\{b,e\},1),(\{b,f\},2),(\{c,f\},5),(\{d,e\},1),(\{e,f\},4)$$

(a) What is the cost of its minimum spanning tree?

(b) How many minimum spanning trees does this graph have?

2. [20 points] Consider the weighted graph with vertex set $\{a,b,c,d,e,f\}$ and weighted edge set

$$(\{a,b\},1),(\{a,d\},1),(\{a,e\},1),(\{b,c\},2),(\{b,e\},2),(\{b,f\},2),(\{c,f\},1),(\{d,e\},1),(\{e,f\},4).$$

(a) Run Kruskal's algorithm on this graph, listing the edges in the order that they are considered (either accepted or rejected). Whenever there is a choice of edges, always use lexicographic ordering. Use conventional set notation to show the disjoint sets at each stage of the algorithm.

You can make this less painful by using a three-column table, the first column listing the weighted edges in the order they're considered, the second column indicating whether the edge is accepted or rejected, and the third column indicating the disjoint sets. Given how the algorithm starts, the table should look like

Edge	Accepted?	Disjoint sets
		$\{a\},\{b\},\{c\},\{d\},\{e\},\{f\}$
$\vdots$	$\vdots$	$\vdots$

(b) Run Prim's algorithm on this graph. Whenever there is a choice of nodes, always use alphabetic ordering (in particular, you'll be starting from node a). Draw a table showing showing the intermediate values of the `cost` array.

3. [10 points] The *maximal spanning tree* of a weighted graph is a spanning tree having maximal weight. Show how to find the maximal spanning tree of a graph.