CISC 5835—Algorithms for Data Analyitcs
Fall, 2019

Chapter 7 Homework

In a linear programming problem, one of three possibilities may occur:

- There may be a unique optimal solution.
- There may be a optimal solution, but it's not unique.
- There may not be a optimal solution.

When asked to identify an optimal solution, make sure to identify which of these three cases occurs.

1. [10 points] Consider the feasible region given by the constraints

$$\begin{aligned} x_1 + 2x_2 &\leq 8 \\ x_1 &\leq 6 \\ x_2 &\leq 2 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \end{aligned}$$

(a) Plot the feasible region.

(b) What are the vertices of the feasible region?

2. [10 points] Identify the optimal solutions of the following linear programming problems, over the feasible region described in Problem 1.

(a) Maximize $3x_1 + 2x_2$.

(b) Maximize $x_1 + 2x_2$.

3. [5 points] For the linear program

$$\text{minimize } 8y_1 + 6y_2 + 2y_3$$

subject to the constraints

$$\begin{aligned} y_1 + 2y_2 &\geq 3 \\ 2y_1 + y_3 &\geq 2 \\ y_1 &\geq 0 \\ y_2 &\geq 0 \\ y_3 &\geq 0 \end{aligned}$$

give the optimal value of the objective function.

Hint: Although this is a 3-variable problem, it is *very* easy to find the optimal value of the objective function. Note that I I am not asking you to find the optimal solution (y_1, y_2, y_3), just the optimal value of the objective function. This can be done with only a tiny amount of calculation.

4. [10 points] Consider the feasible region defined by the constraints

$$\begin{aligned} x_1 + x_2 &\geq 8 \\ x_1 &\geq 2 \\ x_2 &\geq 1 \end{aligned}$$

(a) Plot the feasible region.

(b) What are the vertices of the feasible region?

5. [10 points] Identify the optimal solutions of the following linear programming problems, over the feasible region described in Problem 4.

(a) Minimize $2x_1 + 3x_2$.

(b) Maximize $2x_1 + 3x_2$.

6. [10 points] The Acme Corporation makes giant springs, rideable rockets, and anvils. The following table gives the per-item profit, as well as the manufacturing requirements, of each product (with costs being in dollars and times being in hours):

item	profit	iron (pounds)	packing cost
spring	5	10	3
rocket	10	6	8
anvil	15	5	1

Suppose that we have 200 pounds of iron, and that we can afford only $400 to spend on packing.

(a) Set this up as a linear program, using s, r, and a to denote the number of springs, rockets, and anvils we're going to make. To save sapce, you may write this in matrix-vector form.

(b) Find the optimal choice of r, s, and a, as well as the maximal profit this strategy yields.

Please show your work. Depending on how you solve the problem, this may involve a screen shot.

Suggestion: You'll most likely want to find some auxiliary software to do this.

- Mathematica™ has a built-in `LinearProgamming` function.
- There are packages that solve optimization problems for both C++ and Python.
- You can certainly find web pages that solve linear programs. If you use such a page, please include its URL.