CISC 1400—Discrete Structures
Summer Session I, 2019

PRACTICE MIDTERM EXAMINATION

1. Draw Venn diagrams that illustrate following operations:

   (a) $A \cap B$.

   (b) $(A \cap B)'$.

2. Let

$$A = \{2, 3, 5, 7, 11\}$$
$$B = \{2, 4, 6, 8, 10\}$$
$$C = \{1, 3, 5, 7, 9\}$$

   Determine the following:

   (a) $A \cap B$

   (b) $(A \cap B) \cup C$

   (c) $(A \cup B) \cap C$

   (d) $A - B$

   (e) $|\mathscr{P}(B)|$

3. Consider the sequence

$$1, 9, 17, 25, 33, \ldots.$$

(a) What is the next term in the sequence?

(b) Determine the recursive formula for the sequence. (Don't forget the starting value!)

(c) Determine the closed formula for the sequence.

4. Express the sum

$$3 + 6 + 9 + 12 + 15 + 18$$

using sigma-notation.

5. Evaluate the sum

$$\sum_{i=1}^{5} (3i + 2)$$

6. A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian, and 13 have taken courses in both French and Russian. If 2092 students have taken at least one of Spanish, French, and Russian, how many students have taken a course in all three languages?

7. Use a truth table to prove DeMorgan's Law

$$(p \wedge q)' \equiv p' \vee q'$$

8. Suppose that you (or somebody else) has proved that the propositional equivalence

$$p \wedge (q' \vee r) \equiv (p \wedge q') \vee (p \wedge r)$$

is true. The duality principle tells us that the dual of this equivalence is also true. What is the dual of the equivalence given above?

9. Let $S = \{1,2,3\}$. Consider the relation

$$r = \{(1,1),(1,2),(1,3),(2,3),(3,3)\}$$

on S.

(a) Explicitly state which of the "big five" properties are satisfied by this relation r. If you do not put either a `Y` or a `N` in the blank provided, then that part of the problem will be marked as being incorrect.

- Reflexive? ____
- Irreflexive? ____
- Symmetric? ____
- Antisymmetric? ____
- Transitive? ____

(b) Could r be a function from S to S? Explain why or why not.

10. Use mathematical induction to show that

$$\sum_{i=1}^{n}(4i-3) = 1+5+9+\cdots+(4n-3) = n(2n-1) \qquad \forall n \in \mathbb{Z}^{+}.$$

11. A sequence $a_1, a_2, \ldots$ of numbers is said to be a *Cauchy sequence* if

$$\forall \varepsilon > 0, \exists N \in \mathbb{Z}^* : m \geq N \wedge n \geq N \implies |a_m - a_n| < \varepsilon$$

Use quantification rules to determine a precise statement of what it means for a sequence to *not* be a Cauchy sequence.

Note: You'll need to negate an implication. The easiest way to do this is to use the implication rule

$$[p \implies q] \equiv [p' \vee q]$$

If you let $p(N, \varepsilon)$ be the statement

$$m \geq N \wedge n \geq N \implies |a_m - a_n| < \varepsilon,$$

then you'll be able to save a bit of writing in the intermediate steps.